(18) max of $3^{3/5}$ at $x=3$ $\quad -2 < x \le 3$

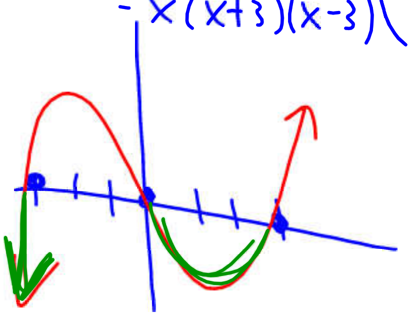
(40) min of 3 at $x=0$
max of 4 at $x=1$

(52) a) No c) No
b) no d) max : $6\sqrt{3}$ at $x=-\sqrt{3}$ and $x=\sqrt{3}$
min : 0 at $x=-3,0,3$

$$f(x) = |x^3 - 9x|$$

$$f(x) = \begin{cases} x^3 - 9x & x \ge 3,\ -3 < x < 0 \\ -x^3 + 9x, & x \le -3,\ 0 < x < 3 \end{cases}$$

$f(x) = x(x^2-9)$
$= x(x+3)(x-3)$



$$f'(x) = \begin{cases} 3x^2 - 9, & x > 3,\ -3 < x < 0 \\ -3x^2 + 9, & x < -3,\ 0 < x < 3 \end{cases}$$

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37) $y = x\sqrt{4-x^2}$ $[-2, 2]$

$y' = x \cdot \frac{1}{2\sqrt{4-x^2}} \cdot (-2x) + \sqrt{4-x^2} \cdot 1$

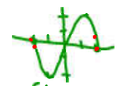
$= \frac{-x^2}{\sqrt{4-x^2}} + \sqrt{4-x^2} \cdot \frac{\sqrt{4-x^2}}{\sqrt{4-x^2}}$

$= \frac{-x^2 + (4-x^2)}{\sqrt{4-x^2}} = \boxed{\frac{-2x^2+4}{\sqrt{4-x^2}}}$

$-2x^2+4=0$
$4=2x^2$
$2=x^2$
$\pm\sqrt{2}=x$

$\sqrt{4-x^2}=0$
$x=2,-2$

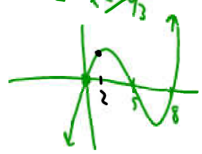
Check:
$f(\sqrt{2}) = 2$ abs. max
$f(-\sqrt{2}) = -2$ abs. min
$f(1.9) > f(2) = 0$ local min
$f(-1.9) < f(-2) = 0$ local max



43) $V(x) = x(10-2x)(16-2x)$ $0<x<5$

$V'(x) = 160 - 104x + 12x^2$
$= 4(x-2)(3x-20)$
$x=2$ $x=\cancel{20/3}$

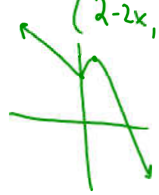
$V(2) = 144$



40) $y = \begin{cases} 3-x, & x<0 \\ 3+2x-x^2, & x\geq 0 \end{cases}$

$y' = \begin{cases} -1, & x<0 \\ 2-2x, & x>0 \end{cases}$

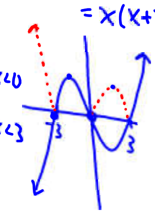
f' undef at $x=0$
$f'=0$ at $x=1$
$f(0) = 3$ local min
$f(1) = 4$ local max



52) $f(x) = |x^3 - 9x|$

$f(x) = |x^3-9x|$
$= x(x^2-9)$
$= x(x+3)(x-3)$

$f(x) = \begin{cases} x^3-9x, & x>3,\ -3<x<0 \\ -x^3+9x, & x<-3,\ 0\leq x<3 \end{cases}$



$f'(x) = \begin{cases} 3x^2-9 \\ -3x^2+9 \end{cases}$

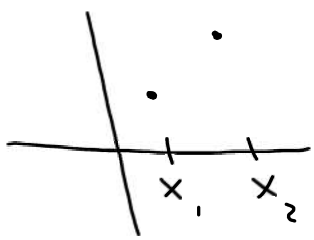
$f'(x)=0$ at $x=\pm\sqrt{3}$
$f'(x)$ undef at $x=-3,0,3$

More 4.2 - MVT

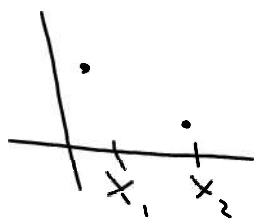
⓪ Better way to determine max/min

For an interval I, let x_1 and x_2 be any 2 points.

- f increases on I if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$



- f decreases on I if $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$



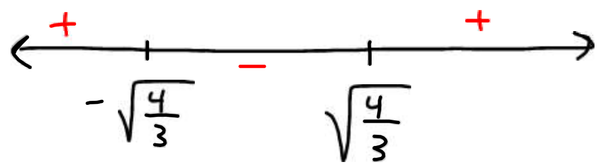
Corollary to MVT

Let f be cont. on $[a,b]$ and diff. on (a,b) then ...

- if $f'(x) > 0$ at each point in $[a,b]$, then $f(x)$ must increase on $[a,b]$
- if $f'(x) < 0$ at each point in $[a,b]$, then $f(x)$ must decrease on $[a,b]$

Ex) Where is $f(x) = x^3 - 4x$ decreasing or increasing?

$f'(x) = 3x^2 - 4$



$f' = 0?$ $\quad 3x^2 = 4$

$x^2 = \frac{4}{3}$

$x = \pm\sqrt{\frac{4}{3}}$

f'	+	−	+
f	inc	dec	inc

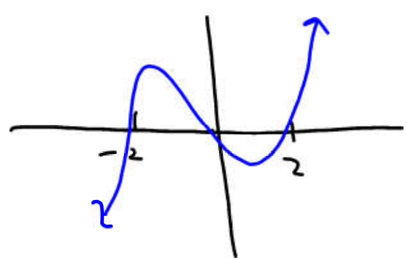
$f(x)$ increasing: $\left(-\infty, -\sqrt{\frac{4}{3}}\right] \cup \left[\sqrt{\frac{4}{3}}, \infty\right)$

$f(x)$ decreasing: $\left[-\sqrt{\frac{4}{3}}, \sqrt{\frac{4}{3}}\right]$

$f(x) = x^3 - 4x$

$= x(x^2 - 4)$

$= x(x+2)(x-2)$



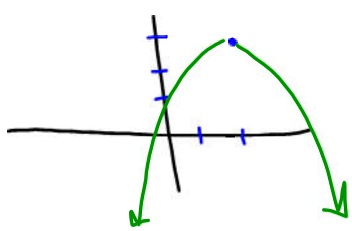
Ex) $f'(x) = 2x$ What could $f(x)$ have been?

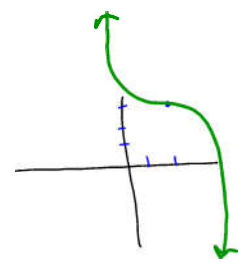
$f(x) = x^2 + C$

TRY: Sketch a graph so that $f(2) = 3$, $f'(2) = 0$

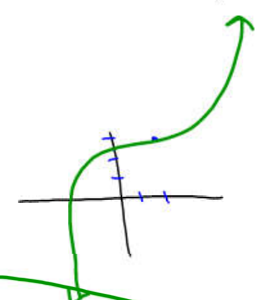
a) $f'(x) \overset{\text{inc}}{>} 0$ for $x < 2$
and $f'(x) \overset{\text{dec}}{<} 0$ for $x > 2$

b) $f'(x) \overset{\text{dec}}{<} 0$ for $x \neq 2$





c) $f'(x) > 0$ for $x \neq 2$



HW: p 202 #1, 15, 17, 24, 25, 29-41 odds